Theoretical Uncertainties in $\mathbf{\Gamma}_{\mathrm{sl}}(\mathbf{b \to u})$

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Abstract

I review the existing theoretical uncertainties in relating the semileptonic decay width in $b \to u$ transitions to the underlying Kobayashi-Maskawa mixing element $|V_{ub}|$. The theoretical error bars are only a few per cent in $|V_{ub}|$, with uncertainties from the impact of the nonperturbative effects nearly negligible.

Inclusive semileptonic decay widths of beauty hadrons offer the theoretically most clean way to determine the underlying KM mixing angles describing the weak couplings of b quark to W boson. For $b \to c$ transitions the semileptonic width is (almost) directly measured in experiment. This allowed one to determine $|V_{cb}|$ in a model-independent way with unprecedented accuracy of only a few per cent [1].

Following the same route in the quest for $|V_{ub}|$ is much more involved experimentally. Direct accurate measurement of $\Gamma_{sl}(b \to u)$ for a long time seemed questionable. The dedicated studies conducted over the last few years suggested a feasibility of such measurements at the competitive level of model-independence. The first results have already been reported [2]. Discussion of some theoretical aspects involved in unfolding the $b \to u$ width from the measurable decay distributions, can be found in Refs. [3].

In this note I review the existing theoretical uncertainties in relating $\Gamma_{sl}(b \to u)$ to $|V_{ub}|^2$. They are rather small, and will be dominated by the uncertainties involved in the experimental determination of $\Gamma_{sl}(b \to u)$ in the foreseeable future.

1 The theoretical framework

The operator product expansion (OPE) applied to the inclusive decay probabilities of a heavy hadron yields the basic expression [4]

$$\Gamma_{sl}^{b \to u}(B) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{ub}|^2 \left\{ A_0 \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} \right) - 2\frac{\mu_G^2}{m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3}\right) \right\}, \qquad (1)$$

with the leading power corrections given in terms of the two expectation values

$$\mu_\pi^2 = \frac{1}{2M_B}\langle B|\bar{b}(i\vec{D})^2 b|B\rangle, \qquad \mu_G^2 = \frac{1}{2M_B}\langle B|\bar{b}\frac{i}{2}g_s\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle \qquad (2)$$

having the transparent physical meaning. The practically most important feature of the OPE result is the absence of the potential $1/m_b$ nonperturbative corrections [4]. They could have been naively expected from the strong dependence of $\Gamma_{sl}(B)$ on the mass, and their potential size could have been as large as 20 to 40% even for b particles. The actual nonperturbative corrections in QCD start with $1/m_b^2$ and hence emerge at the scale of 5%. Therefore, in practical terms they must be included, but can be treated in a simplified fashion relying on the so-called 'practical' version of the OPE in QCD. The leading terms in Eq. (1) representing the partonic width free from bound-state nonperturbative effects, deserves attention to the first place. It includes the purely perturbative corrections embedded into the coefficient function A_0.

1.1 Perturbative corrections

The perturbative expansion of the decay width has the general form

$$\Gamma_{sl}^{pert}(b \to u) = \frac{G_F^2 m_b(\mu)^5}{192\pi^3} |V_{ub}|^2 \left[1 + a_1 \frac{\alpha_s(\tilde{\mu})}{\pi} + a_2 \left(\frac{\alpha_s(\tilde{\mu})}{\pi}\right)^2 + a_3 \left(\frac{\alpha_s(\tilde{\mu})}{\pi}\right)^3 + ... \right].$$
$$(3)$$

The important feature explicit in Eq. (3) is that in the quantum field theory like QCD the masses, similar to all other 'couplings' determining the underlying Lagrangian, are "running" (depend on normalization point). As a result, the perturbative coefficients a_1, a_2, ... depend, in general, on the normalization points μ and $\tilde{\mu}$ for m_b and α_s, respectively.

The perturbative coefficient a_1 is well known from the computations of the QED corrections to the muon decay. The effects associated with running of α_s in the first-order QCD corrections generate the whole series in α_s and they were computed in Ref. [5] to all orders in the generalized BLM [6] approximation. The most challenging used to be the so-called "genuine" non-BLM α_s^2 perturbative corrections, which were recently computed for $b \to u$ in Ref. [7]. The normalization scheme for the heavy quark mass appropriate for the heavy quark expansion was found in Refs. [8, 9]. The two-loop (and the all-order BLM) evolution of such mass was computed in Ref. [10]. Using this low-scale running quark mass was instrumental for the precise numerical evaluation of m_b, Ref. [11].

It is important to note that it is not possible to choose $\mu = 0$ (which would correspond to using the 'pole' mass m_b^{pole} in Eqs. (1,3)) to evaluate the width with a power-like accuracy [12]: this would bring in uncontrollable $1/m_b$ corrections from the infrared domain of momenta $\sim \Lambda_{\text{QCD}}$. As mentioned above, such uncertainties would be at the level of 20% for B decays.

The size of the higher-order perturbative corrections in Eq. (3) is crucial for evaluating the accuracy of our estimates of the semileptonic width. The well-known source of the potentially large higher-order perturbative effects is associated with the running of α_s. Since the leading BLM series has been computed, these effects cannot induce sizeable uncertainties. In the case of semileptonic widths there is a potentially more significant source associated with the large power $n = 5$ of m_b in Γ_{sl}. This dependence can generally lead to the coefficients a_k in the perturbative order k growing as n^k [8]. This is true even if the strong coupling does not run at all. However, the leading subseries of such corrections can be resummed. It reduces to choosing the proper normalization point μ for the mass $m_b(\mu)$ in Eq. (3), which must scale according to $\mu \sim m_b/n$ [8]. Below I illustrate the utility of this large-n resummation on the example of the second-order corrections. This complements the analysis of Ref.[13] dedicated to the BLM corrections.

Since the BLM subseries is known, it is convenient to concentrate on the non-BLM corrections alone which are obtained subtracting the terms of the form $\beta_0^k \left(\frac{\alpha_s}{\pi} \right)^{k+1}$, where $\beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f$ is the first coefficient of the β-function for the strong coupling. (n_f denotes the number of light flavors excluding b quark). This is particularly useful in discussing the second-order corrections since the non-BLM coefficient $a_2^{(0)}$ does not depend on the scale $\tilde{\mu}$ chosen for the strong coupling $\alpha_s(\tilde{\mu})$.

In terms of the pole mass (that is, with $\mu \to 0$) the two non-BLM coefficients $a_1^{(0)}$

and $a_2^{(0)}$ are [7]

$$a_1^{(0)}(0) = -\frac{2\pi^2}{3} + \frac{25}{6} \simeq -2.41 , \qquad a_2^{(0)}(0) \simeq 5.54 . \qquad (4)$$

The literal value of $a_2^{(0)}(0)$ is significant. The analysis of Ref. [8] suggests, however, that the appropriate value of the normalization point μ for the mass $m_b(\mu)$ is $\mu \approx m_b/n = 0.2m_b$. In other words, the perturbative coefficients $a_1^{(0)}(m_b/5)$, $a_2^{(0)}(m_b/5)$, ... must not be enhanced. For illustration we can neglect them altogether, that is, assume they vanish. This would determine the perturbative coefficients in terms of the pole mass via the relation [10]

$$m_b(0) \simeq m_b(\mu) + \left(\frac{16}{9}\mu + \frac{2}{3}\frac{\mu^2}{m_b} \right) \left(\frac{\alpha_s}{\pi} - N_c \left(\frac{\pi^2}{6} - \frac{13}{12} \right) \left(\frac{\alpha_s}{\pi} \right)^2 \right) + \mathcal{O}\left(\left[\frac{\alpha_s}{\pi} \right]^3 , \beta_0^k \left[\frac{\alpha_s}{\pi} \right]^{k+1} \right) . \qquad (5)$$

In this way with $\mu = 0.2m_b$ we arrive at the estimate

$$a_1^{(0)}(0) \simeq -1.91 , \qquad a_2^{(0)}(0) \simeq 4.68 . \qquad (6)$$

The magnitude of the second-order coefficient is reproduced!

Clearly, an accurate match obtained above is partially accidental, and *a priori* one should not have expected such an approximation to work with better than 30% accuracy. It is not possible to rule out using, say, $\mu = 0.2m_b$ or $\mu = 0.3m_b$. In any case, even complete account for the n-enhanced corrections leaves out regular terms, for example, a finite short-distance renormalization of the $\bar{b}\gamma_\alpha(1-\gamma_5)u$ weak current characterized by the momentum scale $\sim m_b$. These are not peculiar to the decay widths and are expected to have perturbative expansions with coefficients of order unity.

Thus, the calculated second-order perturbative corrections are very moderate for m_b normalized at the scale about $m_b/5 \simeq 1\,\text{GeV}$, and the similar behavior is expected from the higher-order effects (first of all, α_s^3 corrections). On the other hand, just this low-scale running mass $m_b(\mu)$ can be accurately determined from the $e^+e^- \to b\bar{b}$ cross section near the threshold [14], with the most stability at $\mu \simeq 1.2\,\text{GeV}$ (for a qualitative discussion see, *e.g.*, Ref. [15], Sect. 3.2).

The value of the low-scale running mass $m_b(\mu)$ is routinely translated to the scale $\mu = 1\,\text{GeV}$. Therefore, we express the width in terms of the b quark mass normalized at this scale:

$$\Gamma_{\text{sl}}^{\text{pert}}(b \to u) \simeq \frac{G_F^2 m_b(1\,\text{GeV})^5}{192\pi^3} |V_{ub}|^2 \left[1 + a_1(1\,\text{GeV})\frac{\alpha_s(\tilde{\mu})}{\pi} + a_2^{(0)}(1\,\text{GeV}) \left(\frac{\alpha_s(\tilde{\mu})}{\pi} \right)^2 + \right.$$

$$\left. + a_2^{\text{BLM}}(1\,\text{GeV}, \tilde{\mu}) \left(\frac{\alpha_s(\tilde{\mu})}{\pi} \right)^2 \right] , \qquad (7)$$

$$a_1(1\,\text{GeV}) \simeq -0.32 , \qquad a_2^{(0)}(1\,\text{GeV}) \simeq -1.28 .$$

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Here $m_b \simeq 4.6\,\mathrm{GeV}$ is assumed.

The value of a_2^{BLM} depends on the scale $\tilde{\mu}$. If $\tilde{\mu} = m_b$ then $a_2^{\mathrm{BLM}} \approx 3.6$ (neglecting the contribution from the $c\bar{c}$ quarks). Identifying the appropriate scale $\tilde{\mu}$ for α_s in the standard $\overline{\mathrm{MS}}$ scheme, it is often advantageous to use its commensurate scale for a more physical coupling, which to our accuracy amounts to using $\tilde{\mu}_{\overline{\mathrm{MS}}} = \mathrm{e}^{-5/6} m_b$. With this choice $a_2^{\mathrm{BLM}}(1\,\mathrm{GeV}, \mathrm{e}^{-5/6} m_b) \simeq 4.8$. In view of a smallish size of the first-order correction in Eq. (7) this has no practical significance, however. The c quark loops are expected to increase the value of a_2^{BLM} by 1 ± 0.5.

1.2 Numerical value of $m_b(1\,\mathbf{GeV})$

The value of $m_b(1\,\mathrm{GeV})$ can be accurately determined from experimental cross section $e^+ e^- \to b\bar{b}$. Even without incorporating any corrections at all, one obtains a reasonable estimate about $4.60\,\mathrm{GeV}$ (see, *e.g.*, [15]). The leading and next-to-leading perturbative corrections are modest and yield an estimate $m_b(1\,\mathrm{GeV}) \simeq 4.50$ to $4.55\,\mathrm{GeV}$, depending on the value of α_s. The NNLO determination was performed recently [11] resulting in $m_b(1\,\mathrm{GeV}) \simeq 4.56\,\mathrm{GeV}$, with the stated uncertainty of about $50\,\mathrm{MeV}$. There are certain directions along which this estimate can be refined in the future. On the other hand, a number of considerations suggest that this value is on the lower side and most probably represents the lower bound for the possible values of m_b. To be conservative, I shall assign the uncertainty $60\,\mathrm{MeV}$ to m_b:

$$m_b(1\,\mathrm{GeV}) \;=\; 4.58 \pm 0.06\,\mathrm{GeV}\;. \tag{8}$$

This value refers to the specific physical scheme (the "kinetic" mass) defined in Refs. [8, 10].

2 Nonperturbative effects

2.1 Power corrections

The $1/m_b^2$ nonperturbative corrections decrease the width by approximately -4%. The chromomagnetic expectation value is estimated through the hyperfine splitting

$$\mu_G^2 \;\simeq\; \frac{3}{4}\left(M_{B^*}^2 - M_B^2\right) \;\simeq\; 0.4\,\mathrm{GeV}^2\;, \tag{9}$$

with the conservative estimated accuracy $\pm 25\%$ reflecting the intrinsic $1/m_b$ effects and the perturbative corrections in the coefficient function related to the complete field-theoretic definition of this operator.

The kinetic expectation value $\mu_\pi^2(\mu)$ traditionally is evaluated at the scale $\mu \simeq 0.7\,\mathrm{GeV}$ and is somewhat uncertain at present. The inequality $\mu_\pi^2(\mu) > \mu_G^2(\mu)$ [16, 17, 18] essentially limits the range of its possible values; the critical review can be

found in Ref. [1]. Since the dependence of $\Gamma_{\mathrm{sl}}^{b \to u}$ on μ_π^2 is weak, we adopt here an overly conservative range

$$\mu_\pi^2 = (0.6 \pm 0.2) \ \mathrm{GeV}^2 \ . \tag{10}$$

At order $1/m_b^3$ the nonperturbative effects can show up as the $1/m_b$-suppressed pieces of the kinetic and chromomagnetic operators, which are not expected to exceed the 25% level of their leading-order contributions accessed above. There are also two new operators, one of them leading to the so-called Darwin term

$$\rho_D^3 = -\frac{1}{2M_B} \langle B| \frac{g_s^2}{2} \bar{b} \gamma_\alpha t^a b \sum_q \bar{q} \gamma_\alpha t^a q |B\rangle \ . \tag{11}$$

The second operator is of a similar four-fermion form but includes only the u light quark and has a different color and Lorentz structure:

$$\frac{1}{2M_B} \langle B|\bar{b}\gamma_\alpha(1-\gamma_5)u \, \bar{u}\gamma_\beta(1-\gamma_5)b|B\rangle (\delta_{\alpha\beta} - v_\alpha v_\beta) \ , \qquad v_\mu = \frac{P_\mu^B}{M_B} \ . \tag{12}$$

The later operator generates the 'spectator-dependent' corrections sensitive to the flavor of the light antiquark in B meson. The expectation value Eq. (12) describes the effect of weak annihilation (WA) and in general differentiates $\Gamma_{\mathrm{sl}}^{b \to u}(B^-)$ from $\Gamma_{\mathrm{sl}}^{b \to u}(B^0)$; the Darwin operator is an isosinglet and affects the widths uniformly. The above operators emerge at the momentum scale governed by m_b and must be evolved down to the hadronic scale. They mix under renormalization, and different color and Lorentz structures appear.

With massless leptons the effect of WA vanishes in the factorization approximation, and is expected to be dominated by the nonfactorizable piece [19]. Therefore, it must be suppressed. The coefficient of the Darwin operator was first evaluated in Refs. [20, 21] (see also [22]), and its expectation value can be estimated by factorization [16]. The detailed discussion can be found in the dedicated paper [23], with the final estimate

$$\frac{\delta\Gamma_{\mathrm{sl}}^{\mathrm{Darwin}}(b \to u)}{\Gamma_{\mathrm{sl}}(b \to u)} \approx -(1 \div 2)\% \ . \tag{13}$$

To get an independent idea of the size of nonfactorizable contributions, we can use data on D decays. In the estimates, we employ the following ingredients as input:
- $SU(3)$ symmetry
- WA effect in $\Gamma_{D_s} - \Gamma_{D^0}$
- Nonperturbative effects in $\Gamma_{\mathrm{sl}}(D)$.

More specifically, we attribute a significant part of the excess in $\Gamma_{\mathrm{sl}}(D)$ compared to the OPE estimate, to the effect of the nonfactorizable four-fermion expectation value.[2] We then arrive at the following evaluation of the isosinglet and isotriplet

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[2]The relation of this assumption to the possible duality violation will be elucidated elsewhere.

effects, respectively:

$$
\begin{aligned}
\frac{\delta_s \Gamma_{\text{sl}}(b \to u)}{\Gamma_{\text{sl}}(b \to u)} &\equiv \frac{\delta \Gamma_{\text{sl}}^{\text{nf}}(b \to u)|_{B^-} + \delta \Gamma_{\text{sl}}^{\text{nf}}(b \to u)|_{B^0}}{2\Gamma_{\text{sl}}(b \to u)} \approx 0.04 \, \frac{\delta \Gamma_{\text{sl}}^{\text{nf}}(D^0)}{\Gamma_{\text{sl}}(D^0)} \\
\frac{\delta_{\text{WA}} \Gamma_{\text{sl}}(b \to u)}{\Gamma_{\text{sl}}(b \to u)} &\equiv \frac{\delta \Gamma_{\text{sl}}^{\text{nf}}(b \to u)|_{B^-} - \delta \Gamma_{\text{sl}}^{\text{nf}}(b \to u)|_{B^0}}{\Gamma_{\text{sl}}(b \to u)} \approx 0.05 \, \frac{\Gamma_{D_s}^{\text{WA}} - \Gamma_{D^0}^{\text{WA}}}{\Gamma_{D^0}} \, .
\end{aligned}
\tag{14}
$$

Here we have used the computations of Ref. [23] and its estimates of the so-called color-straight expectation values. The effects safely below a per cent level are discarded.

According to the analysis of Ref. [26], the missing fraction of the D semileptonic width constitutes about 50%. (This corresponds to a small nonfactorizable piece in the expectation value, $g_s \sim 0.02$ [19].) Keeping in mind that m_c is not much larger than the hadronic scale, it is reasonable to adopt

$$
\frac{\delta \Gamma_{\text{sl}}^{\text{nf}}(D^0)}{\Gamma_{\text{sl}}(D^0)} \approx 0.25 \text{ to } 0.5 \, .
$$

The analysis of the second Ref. [19] suggests that the major origin of difference between τ_{D_s} and τ_{D^0} comes from WA, while other effects, *e.g.* related to $SU(3)$ breaking probably do not exceed $\sim$5%. Therefore, using $\tau_{D_s}/\tau_{D^0} \simeq 1.20$ [24] we assess

$$
\begin{aligned}
\frac{\delta_s \Gamma_{\text{sl}}(b \to u)}{\Gamma_{\text{sl}}(b \to u)} &\approx (1 \div 2)\% \\
\frac{\delta_{\text{WA}} \Gamma_{\text{sl}}(b \to u)}{\Gamma_{\text{sl}}(b \to u)} &\approx -1\% \, .
\end{aligned}
\tag{15}
$$

The isosinglet enhancement of the width tends to offset the effect of the Darwin operator.

The literal application of the $1/m_Q$ expansion in decays of charmed mesons is questionable and is at best semiquantitative. Therefore, we view the above computation rather as an evaluation of the significance of the potential contributions. We will also allow for a factor of 2 increase in the effects, to have a more confident assessment of the related uncertainties. Let us recall that studying the $b \to u$ decay distributions for charged and neutral B *separately* will provide an important information on the nonfactorizable effects in heavy mesons [19].

Thus, the nonperturbative effects in $\Gamma_{\text{sl}}(b \to u)(B)$ computed in the OPE are expected to be about -5%, and can be reliably estimated.

2.2   Violations of local duality

The predictions based on the practical applications of the OPE applied to most observables in the Minkowski space, first of all decay rates, to a certain extent rely on local quark-hadron duality. Although the conceptual origin of its possible violation at finite energies has been clarified in recent studies [25, 26], the reliable dynamic

evaluation of its significance at intermediate energies still lies beyond the possibilities of modern theory. Can one expect duality violations to affect credibility of determination of $|V_{ub}|$ from $\Gamma_{\rm sl}^{b \to u}(B)$?

For the OPE-amenable observables violation of local duality is intrinsically related to the asymptotic nature of the power expansion in QCD. This means that at finite m_b including higher and higher terms in $1/m_b$ – even if they all were known – would improve the accuracy of the predictions only up to a point. This perspective may suggest *a priori* an optimistic viewpoint that the duality violation is safely below the effect of the nonperturbative corrections which have been evaluated. While this is the most natural assumption which often holds, it must not necessary be true, as can be traced in certain model considerations.

Nevertheless, there are sound reasons to believe that local duality violation in $\Gamma_{\rm sl}^{b \to u}(B)$ should not be noticeable at a per cent level relevant in practice. Some of the arguments can be found in the lectures [15], Sect. 3.5.3 and rely on the general constraints the duality-violating effects must obey, on the one hand, and on the experimental information on other hard processes in QCD at intermediate energies, most notably the resonance physics. The key fact is that the energy release is large enough, so that a significant number of channels are open even if the resonance structures are not yet completely washed out in a particular channel [26]. The dynamical models of duality violations typically predict negligible effects at the mass scale around $5\,{\rm GeV}$.

The violations of local duality in the decay widths was recently considered in the framework of the exactly solvable 't Hooft model – $1+1$ dimensional QCD in the limit of a large number of colors. This model exhibits in full the part of the actual QCD phenomenology which is expected to play a crucial role in violation of local duality, *viz.* quark confinement and manifest resonance dominance. The analytic studies in Refs. [27, 28] led to the conclusion that for the actual mass of the b quark the duality-violating effects must lie below a percent level. The similar conclusion can be drawn from the numerical studies of Refs. [29] viewed from the proper perspective.

Of course, QCD in $1+1$ dimensions cannot fully represent ordinary QCD. In particular, it misses to incorporate the possible effects of transverse gluons absent in two dimensions. In this respect, it is important to keep in mind that the overall effect of the perturbative corrections in $\Gamma_{\rm sl}^{b \to u}$ in the proper OPE approach does not exceed a 10% level, see Eq. (7), so that even a delayed onset of duality here can hardly bring in a significant effect.

To summarize, one expects duality violation to be negligible in $\Gamma_{\rm sl}(b \to u)$.

2.3   Comments on the literature

Since the development of the dynamic $1/m_Q$ expansion for the inclusive widths, suggestions surface every now and then in the literature which challenge applicability of the OPE in one form or another. For example, paper [30] claimed identifying a certain class of 'kinematic' nonperturbative effects which are allegedly missed in the conventional OPE approach, and must be incorporated additionally. This was ap-

plied to the semileptonic $b \to u$ width in Ref. [31] and resulted in an essentially larger positive nonperturbative corrections. It thus seems appropriate to dwell on this issue in the context of the present note.

Unfortunately, there is certain misleading element in that the approach of Refs. [30, 31] is claimed to be derived from the first principles of QCD. This is not so in reality, and simply cannot be since the result is incompatible with a few very general properties. In particular, this refers to Eq. (2) of Ref. [31] which is the starting expression for the width. In fact, this is simply a parton model motivated ansatz employing only the heavy quark analogue of the leading-twist distribution function. While the leading-twist distribution function captures properly the major effects of the so-called "Fermi motion" [32] on the decay *distributions*, it cannot be responsible for the corrections to the integrated rates which start explicitly only with the higher-twist effects.

The model ansatz of Ref. [31] is adjusted to correctly reproduce the absence of $1/m_b$ corrections to the inclusive width. Its deficiency nevertheless manifests itself already at the order $1/m_b^2$ where the first nonperturbative corrections to the width appear. Indeed, it is easy to obtain what is the $1/m_b$ expansion of the ansatz, using Eqs. (5) and (6) of Ref. [31]:

$$\Gamma_{\mathrm{sl}}^{\mathrm{Ref.}[31]}(B) \;=\; \Gamma_{\mathrm{sl}}^{\mathrm{parton}} \left[1 + \frac{35}{6} \frac{\mu_\pi^2}{m_b^2} - \frac{5}{2} \frac{\mu_G^2}{m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3} \right) \right] . \tag{16}$$

The comparison with the OPE result Eq. (1) shows that neither the chromomagnetic nor the kinetic operator contributions are reproduced. The most dramatic difference appears in the latter: the coefficient for μ_π^2 is almost 12 times larger and has the opposite sign!

It is a simple matter to see which of the two expressions is correct, and this does not require going through the whole machinery of the OPE in QCD. It suffices to look at the decay rate of a *free* quark moving with the small velocity $\vec{v} \sim 1/m_b$. Its spacelike momentum is then $\vec{p} = m_b \vec{v}$, and the decay rate is simply suppressed by the Lorentz dilation factor $(1 + \vec{p}^2/m_b^2)^{-1/2} \simeq 1 - \vec{p}^2/2m_b^2$. This is the meaning of the corresponding OPE correction in Eq. (1) [19, 16]. Clearly, Eq. (16) stands no chance to hold for B mesons if it fails so heavily even for a free particle.

As a matter of fact, there is a nontrivial example of a theory where the semileptonic decay width of the strongly interacting confined b quark can be calculated analytically and confronted to the $1/m_b$ expansion derived in the OPE [27]. This is the 't Hooft model mentioned in the previous section. The theoretical computations are usually compounded by the necessity to evaluate in parallel the perturbative corrections to the width. However, in the special case of vanishing lepton masses, the perturbative corrections in the 't Hooft model can be computed to all orders in perturbation theory [27]. In this case the decay width of the B mesons, both in $b \to u$ and $b \to c$ transitions, was shown to coincide with its OPE expansion at all computed orders in $1/m_b$. Incidentally, the exact expression for the width in terms of the corresponding light-cone wavefunction of the B meson would have an essentially *different* functional form from the ansatz postulated in Ref. [31].

Failing to comply QCD already at the level of the leading nonperturbative effects, the ansatz of Ref. [31] intrinsically contains large $1/m_b^3$ and higher-order terms, which likewise have nothing to do with actual strong interactions. At the same time, it offers no room for the actual $1/m_b^3$ effects, the potential spectator-dependent contributions from WA which have the transparent underlying origin.

Considering the above facts, we have to conclude that in what concerns the integrated rates, the approach of Refs. [30, 31] is fundamentally flawed, and its numerical outcome cannot be used even to get model-dependent insights into possible theoretical uncertainties associated with accounting for the bound state and hadronization dynamics.

3   Summary and conclusions

Assembling all pieces together, we evaluate the present theoretical predictions for $\Gamma_{\rm sl}^{b \to u}(B)$ as follows:

$$\Gamma_{\rm sl}^{b \to u}(B) = 66 {\rm ps}^{-1} |V_{ub}|^2 \left[1 + 0.065 \frac{m_b(1\,{\rm GeV}) - 4.58\,{\rm GeV}}{60\,{\rm MeV}} \pm 0.02_{\rm pert} \pm 0.035_{\rm nonpert} \right], \tag{17}$$

where the last term lumps together the uncertainties in accounting for the nonperturbative effects. The largest source of uncertainty remains in the precise value of the running b quark mass, although it is only a few per cent. For $|V_{ub}|$ itself we then arrive at

$$|V_{ub}| = 0.00442 \left(\frac{{\rm BR}(B^0 \to X_u \ell \nu)}{0.002} \right)^{\frac{1}{2}} \left(\frac{1.55\,{\rm ps}}{\tau_{\rm B}} \right)^{\frac{1}{2}} \cdot \left(1 \pm 0.025_{\rm QCD} \pm 0.035_{m_b} \right) \, . \tag{18}$$

The uncertainty in μ_π^2 does not affect the theoretical predictions at an appreciable level. No significant uncertainty is expected through the uncalculated higher-order perturbative effects. Some variation, in principle, can be allowed for from the precise value of the strong coupling α_s at a few GeV scale. While the Z-peak physics seems to yield a larger value of α_s, certain low-energy phenomenology would favor a lower value of $\Lambda_{\rm QCD}$. Therefore, it may be premature to rely on the larger value of α_s often applied to low-scale physics, see Ref. [33]. The literal dependence on the value of α_s of the width in Eq. (7) is very weak, so it cannot be used to estimate the overall uncertainties in the perturbative corrections.

The current estimated value of $|V_{ub}|$ is close to the original evaluation made in Ref. [13].[3] This is not accidental, for both the progress in the determination of m_b and the state of the art computations of the second-order corrections yielded very moderate effects if one relies on the proper OPE-compatible low scale mass m_b, as in Ref. [13]. Such a stability is a good sign indicating that the theoretical relation between $\Gamma_{\rm sl}^{b \to u}(B)$ and $|V_{ub}|^2$ rests on sound grounds.

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[3]Some numerical difference is related to using there a lower value of the quark mass.

It is often tempting to determine the ratio of the KM mixing angles $|V_{ub}/V_{cb}|$. When extracted from the decay widths, $|V_{ub}|$ and $|V_{cb}|$ often share common theoretical uncertainties which can partially cancel in the ratio. For example, this happens with the dependence on the exact value of m_b. Nevertheless, it is important to keep in mind that even the underlying problems are not always identical. The energy release in $b \to u$ is safely large to ensure a good control of the nonperturbative effects, without a recourse to the heavy quark symmetry. A more limited energy release in the decays $b \to c$ makes it *a priori* more vulnerable to possible effects of duality violation, sensitive to the structure of the higher-order nonperturbative corrections and to applicability of the heavy quark symmetry to charm particles through the value of $m_b - m_c$. On this route the dominant dependence on μ_π^2 emerges for $|V_{cb}|$. All these theoretical ingredients will be critically examined when the new generation of experimental data on B decays become available.

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